VOLUME 79 SEPARATE No. 295

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

OCTOBER, 1953

DISCHARGE CHARACTERISTICS OF TAINTER GATES

by Arthur Toch, J.M. ASCE

Presented at
New York City Convention
October 19-22, 1953

HYDRAULICS DIVISION

(Discussion open until February 1, 1954)

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

THIS PAPER

--represents an effort of the Society to deliver technical data direct from the author to the reader with the greatest possible speed. To this end, it has had none of the usual editing required in more formal publication procedures.

Readers are invited to submit discussion applying to current papers. For this paper the final closing dead line appears on the front cover.

Those who are planning papers or discussions for "Proceedings" will expedite Division and Committee action measurably by first studying "Publication Procedure for Technical Papers" (Proceedings — Separate No. 290). For free copies of this Separate—describing style, content, and format—address the Manager, Technical Publications, ASCE.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

DISCHARGE CHARACTERISTICS OF TAINTER GATES

by

Arthur Toch

Iowa Institute of Hydraulic Research
State University of Iowa
Iowa City

Synopsis

Although Tainter gates are among the most widely used means for the control of water, little has been known about their discharge characteristics. Two methods are therefore presented which can be used to describe these characteristics. The discharge coefficient of the Tainter gate for free and submerged flow is shown in its relationship to parameters expressing the gate geometry. With the aid of a simplified theoretical analysis, the energy loss occurring during submerged flow is depicted in a similar manner. The methods applied herein to Tainter gates on level floors should be applicable to other gate shapes.

Introduction

Prediction of the efflux characteristics of Tainter gates in the course of their design has been possible in the past only through specific model studies. The present paper is intended to present general methods for their evaluation which should prove of use in the design and operation of gates.

Tainter gates belong to the general class of devices for the control of water which also includes sluice gates and roller gates. A short survey of previous studies of this class is presented here as an introduction to indicate the extent of their applicability.

Efflux from two-dimensional slots has sometimes been linked with efflux from gates. Thus the coefficient of contraction for slots, i. e., the ratio of limiting jet thickness to slot height, derived by von Mises [1] has been considered applicable to sluice gates. Von Mises' analysis contains the usual assumptions of the free-streamline theory of hydrodynamics: constant velocity along the free streamline and constant pressure across the jet at infinity. The computed contraction coefficient varies widely with the ratio of gate opening to headwater depth. The extent to which this analysis applies to real gate efflux can only be determined by comparison to experimental data.

A mathematical treatment of sluice-gate efflux was presented by Pajer [2]. In his analysis the velocity along the free streamline is assumed to vary with elevation. This premise contains, implicitly, the concept of hydrostatic pressure distribution in the sections of uniform flow. Variation of the coefficient of contraction with the ratio of gate opening to headwater depth is small as compared to the large variation obtained for slots. Pajer shows also that the analytically derived jet profiles agree with experimental measurements. The values of the contraction coefficient determined from actual flow are, of course, always larger than the corresponding computed values for irrotational flow, because the jet geom-

etry is modified by the decelerative action of the boundary shear. Moreover, contraction coefficients derived from the analysis are based on the section of uniform flow at infinity, whereas continued boundary resistance causes the actual vena contracta to be located only a short distance from the gate.

Pajer's mathematical treatment of efflux from a sluice gate can be considered only as applying to the limiting condition of a Tainter gate with infinite radius and 90-degree lip angle. Application of the free-streamline theory to the curved surface of the Tainter gate is unfortunately not feasible because of the mathematical difficulties usually involved in treating any but straight solid boundaries. Hydrodynamic theory could be used in the form of Southwell's relaxation method to obtain theoretical solutions for the curved Tainter gate. This method is extremely tedious, however, and each solution would be applicable only to the particular geometric proportions chosen.

Because of the limitations imposed by the assumptions of irrotationality [3], any such application of hydrodynamic theory must be confined to free-jet efflux, for which the tailwater does not interfere with the jet emanating from the gate. Under conditions of submerged efflux, for which the tailwater depth is greater than the sequent depth at which the jet would form a hydraulic jump, an approximation of the actual flow can be accomplished by use of equations expressing continuity and impulse-momentum, and a modification of the Bernoulli equation. This method is given in some detail on a later page. Similar derivations have been published [4], but not in readily

applicable form.

Horton [5] proposed another method of determining the free-jet efflux from Tainter gates. Recurrent references indicate the advisability of including it in this review. In his method a constant multiplier was to be used to convert "theoretical" to actual discharge coefficients. The multiplier was obtained by comparing a limited number of experimentally determined discharge coefficients with the corresponding "theoretical" coefficients. The experimental coefficients were apparently obtained by using a sluice gate with a beveled lip. The equation by means of which the "theoretical" coefficients are to be calculated was written by Koch and Carstanjen [6] for contraction coefficients and based on several approximations which are far removed from actual flow conditions. Any agreement between coefficients derived from actual Tainter-gate discharge and the "theoretical" coefficients is, therefore, fortuitous.

Experimentally, the Tainter gate has undergone considerable scrutiny. Many small-scale tests have been made, often on models for particular projects. The usual efflux equation was generally employed and the attempt made to find that definition of head which would give minimum variation of the discharge coefficient. Agreement has never been reached on which definition of head - total depth, depth to mid-height of opening, or another - should be used for best results. This disagreement can be resolved by noting that all such definitions stand in simple geometric relation to each other and that none is simply significant over the entire geometric range.

Such lack of coordinated knowledge regarding Tainter-

gate characteristics led to the writing of this paper. In the following pages a systematic graphical representation of the relationship among the pertinent variables is indicated. An approximate analysis, based on a few assumptions, permits the energy changes for submerged efflux to be assessed.

Experimental Procedure

To permit generalization of the variety of field installations of Tainter gates, the laboratory studies were made on simply designed models at the Iowa Institute of Hydraulic Research. The first of these [7] consisted of a gate and pier simulating a specific project [8]. The gate, with a radius of 2.17 feet, had a fixed trunnion height of 1.42 feet. The gate opening of the model was adjustable like that of the prototype. In the second model [9] the pier was eliminated, but the trunnion height of the gate was made adjustable. The radius of this gate was 1 foot. Both models were located in laboratory flumes of rectangular cross-section with level floors, 30 inches and 12 inches wide, respectively.

Water was supplied from the circulation system of the laboratory, and the quantity of flow was measured by means of calibrated elbow meters and air-water manometers. The depths of flow upstream and downstream from the gates were measured by point gages and by manometers connected to piezometric openings in the floors of the flumes.

Both series of experiments were made in the following systematic manner. For a given gate opening and discharge, the tailwater elevation was varied, with the headwater free to reach a corresponding depth. In the first series, several gate openings, each with a number of discharges,

were thus tested. Similar runs were made in the second series of experiments, with the additional variation of the trunnion height.

Discussion of Results

General Considerations

As previously mentioned, two different states of efflux can occur in the flow of water from a gate. For free-jet flow, loss of energy in passage through the gate opening is small. For submerged efflux, a considerable part of the kinetic energy of the flow is dissipated in diffusion directly downstream from the gate. The transition from free-jet to submerged flow can be pictured by assuming a hydraulic jump to approach the gate from downstream. The characteristics of the free-jet efflux will obtain until the toe of the jump reaches the gate. As soon as this position is reached, an increase in the headwater depth occurs for a constant rate of flow. The headwater depth continues to increase as the tailwater level rises, and the water surface just below the gate also undergoes a change. From the state of turbulent aeration of the jump at initial submergence, the flow attains a progressively smoother and more nearly level surface as the depth of flow increases, until aeration becomes negligible and the surface elevation at the gate is but little lower than the subsequent level of the tailwater. These visible changes are evidently accompanied by a modification of the form of diffusion. As the depth of submergence increases and aeration becomes less noticeable, the energy of the jet is dissipated with ever less production of surface waves.

The overall problem of gate efflux can be considered

as a function of the geometry alone despite the fact that the complex mechanism of diffusion is involved in it. Two distinct methods of describing the efflux in terms of the geometric proportions of the gate and the flow are presented in the following sections.

For purposes of clarity, a definition sketch, Fig. 1, is included, showing the variables which are of primary

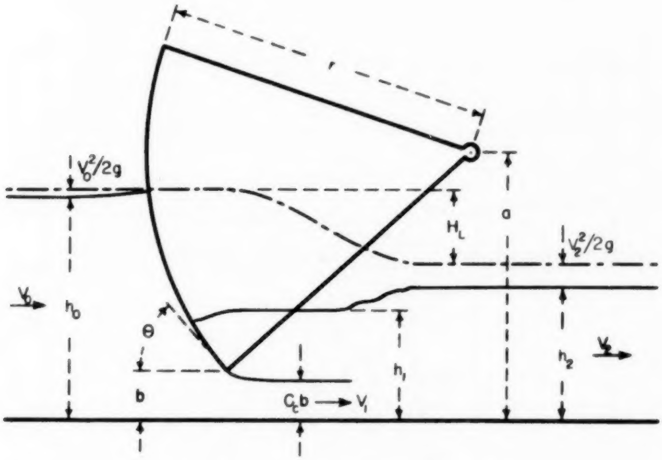

Fig. 1. Definition sketch

importance in the study of the Tainter gate. Other variables are defined as they are introduced. Mention should also be made here that the loss of head in the uniform flow sections upstream and downstream from the gate is ignored. Assumptions are introduced wherever they are necessary for the logical development of the discussion. Two-dimensional flow only is treated in this paper, except for the inclusion of contraction coefficients obtained from the first model study.

Systematized Characteristics

Metzler [10] showed that a discharge coefficient for

the Tainter gate could be graphically related to the ratios of various pertinent boundary lengths. The coefficient was defined as

$$C_d = \frac{q}{b\sqrt{2gh_o}} \tag{1}$$

in which q is the volume rate of flow per unit width of gate. An extension of Metzler's graph was obtained by the writer through variation of the trunnion height, and the

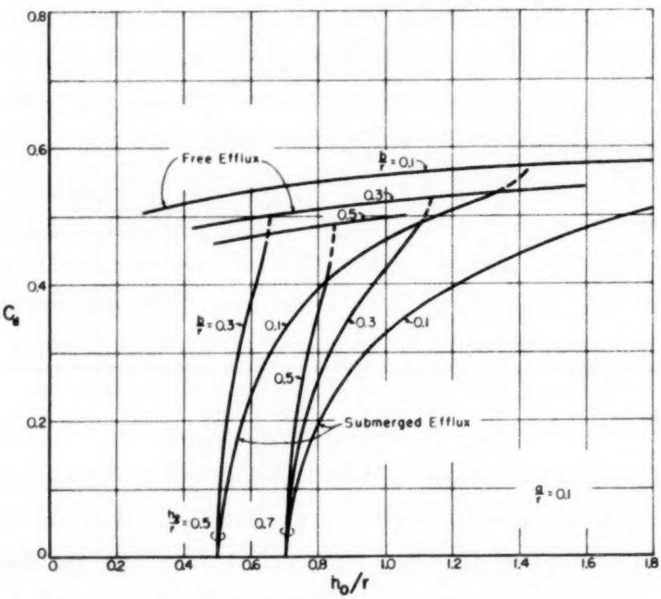

Fig. 2. Coefficient of discharge for free and submerged efflux, a/r = 0.1

results are shown in Figs. 2, 3, and 4. Herein the gate radius r is used as reference, and the basic parameters are thus the relative headwater and tailwater depths h_0/r and h_2/r, the relative height of opening b/r, and the relative trunnion height a/r.

In these figures, the curves rising from the axis of $C_d = 0$ pertain to submerged efflux. The limiting curves on the upper parts of the figures pertain to free efflux. Each figure contains the curves for only one ratio of a/r but for several values of the other parameters. It is evident that for free-jet efflux the changes which occur in the magnitude of the discharge coefficient are relatively small, although the headwater ratio is varied

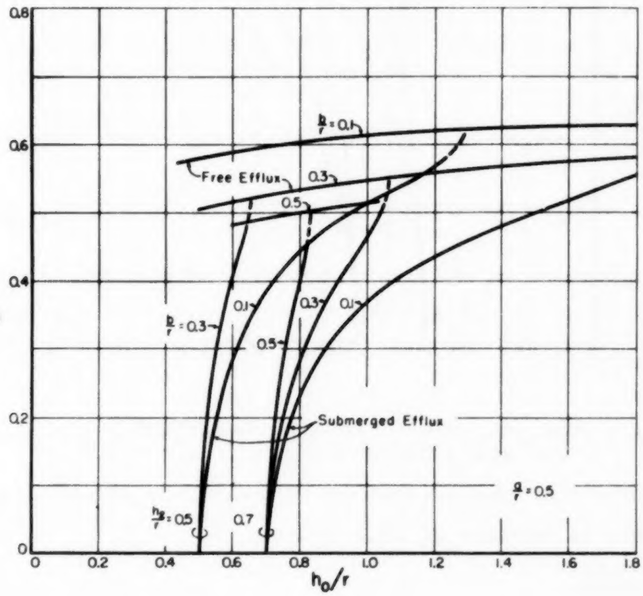

Fig. 3. Coefficient of discharge for free and submerged efflux, $a/r = 0.5$

widely. For submerged flow, however, a wide variation in C_d occurs for the same change in h_0/r . The influence of the other parameters is also apparent in these graphs.

If the discharge corresponding to given numerical values of the geometric variables is to be obtained, the

appropriate ratios can be formed and C_d found from the graphs. Equation (1) will permit calculation of the discharge to be expected. Additional curves, for more numer-

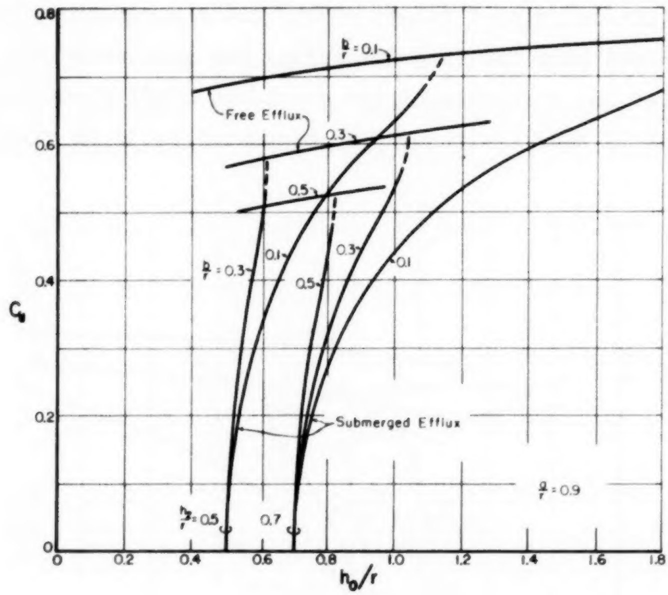

Fig. 4. Coefficient of discharge for free and submerged efflux, $a/r = 0.9$

ical values of the parameters represented, are needed for full use to be made of this method.

<u>Free Efflux</u>

An essential part of the discharge analysis is the evaluation of the lip-angle effect upon the jet contraction. For free efflux the lip angle (Fig. 5) can be seen to modify the coefficient of contraction considerably. As is to be expected, the coefficient increases with decreasing lip angle because the free streamline leaves the lip of the gate in a more nearly horizontal direction. The

scatter of the experimental points, partly the result of the difficulty in measuring accurately the depth of the jet, is undoubtedly also caused by the effect of b/h_0. That at least some variation of C_c with b/h_0 should

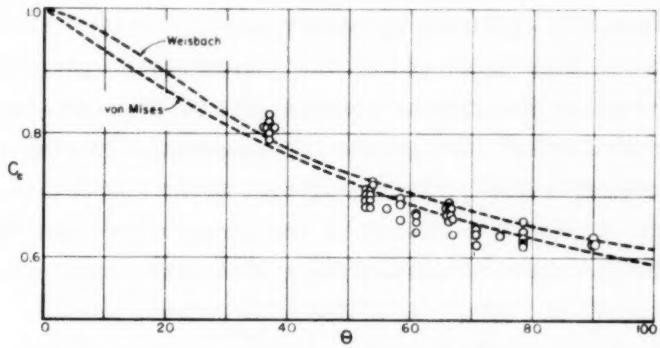

Fig. 5. Effect of lip angle on the coefficient of contraction

occur can be deduced from the variation in the contraction coefficient obtained by Pajer for the sluice gate. But the lack of experimental evidence is not surprising, because the variation can be assumed to be no more pronounced for the Tainter gate than for the sluice gate.

If the discharge coefficient defined by Eq. (1) is to be evaluated from the coefficient of contraction, recourse must be had to the equations of continuity and energy. The resulting relationship,

$$C_d = \frac{C_c}{\sqrt{1 + C_c\, b/h_0}} \qquad (2)$$

if correct, should yield values of C_d in agreement with those obtained from Eq. (1). Corresponding values obtained from Eq. (2) using representative values from the plotted points in Fig. 5, indicate a maximum difference of

about 10 percent in C_d for the present data. If the relationship between C_c and b/h_0 can be clearly determined, the agreement should be better. Some difference will always exist, because the assumption of constant energy implicit in Eq. (2), though approximately correct, is not exactly satisfied in real flow.

For reasons mentioned in the introduction, analytical values for the contraction coefficients of Tainter gates are not available. Two curves for somewhat different flow cases are, therefore, plotted in Fig. 5 for comparison. The curve marked "von Mises" is the locus of the contraction coefficients calculated for a slot with straight inclined walls at $b/h_0 = 0$. The other curve, marked "Weisbach", similarly locates experimental contraction coefficients for efflux from a slot at presumably very small b/h_0 values. The data for both of these curves were obtained from reference [1].

If the axis of symmetry of these slots is assumed to be replaced by a solid surface, a boundary geometry comparable to that of a gate results. At 90 degrees the gate would be equivalent to a sluice gate, and at any angle it could be considered a Tainter gate with infinite radius. Pajer showed that the theoretical coefficient of contraction of the sluice gate departs little from 0.611 for any values of b/h_0 below 0.5. The assumption can hence be made that the contraction coefficient for the slot at $b/h_0 = 0$ will apply, as a first approximation, to the Tainter gate at any angle. Although the curves on Fig. 5 seem to bear out this assumption, a hydrodynamic analysis of the Tainter gate, if mathematically feasible, would still be of considerable aid in determining the actual contraction

coefficients.

Submerged Efflux

If submergence of the gate opening occurs, discharge coefficients computed by means of Eq. (2) for use in Eq. (1) are obviously not applicable. That C_d for submerged flow has a wide range of values was shown by the systematized representation in Figs. 2, 3, and 4. Through use of some simplifying assumptions, however, this efflux condition can be analyzed with fair approximation. The effect of the various geometric parameters is thereby more clearly indicated.

The continuity relationships,

$$q = V_0 h_0 = V_1 C_c b = V_2 h_2 \qquad (3)$$

the energy equations,

$$\frac{V_0^2}{2g} + h_0 = \frac{V_1^2}{2g} + h_1 = \frac{V_2^2}{2g} + h_2 + H_L \qquad (4)$$

and the momentum equation,

$$\frac{1}{2} g \left(h_2^2 - h_1^2\right) = q \left(V_1 - V_2\right) \qquad (5)$$

can be combined to obtain theoretical relationships for the gate efflux. Equations (3) and (5) express, of course, the premises of continuity of flow and of impulse-momentum. Equations (4) are based on the assumption that all loss of head occurs in the zone of diffusion between sections 1 and 2. In Eqs. (4) and (5) the assumption is introduced that hydrostatic pressure distribution prevails not only in the tranquil flow sections but also at section 1 immediately downstream from the gate. The equations are, furthermore, written in terms of mean velocities, with the

implication that the velocity is constant across each section.

Equations (3) and (4) can be solved simultaneously for the discharge,

$$q = h_2\sqrt{\frac{2g(h_0-h_2)}{\Psi+1-(h_2/h_0)^2}} \tag{6}$$

in which $\Psi = H_L/(V_2^2/2g)$. To obtain an expression for the commonly used discharge coefficient C_d , the discharge q is eliminated from Eqs. (1) and (6), so that

$$C_d = \frac{h_2}{b}\sqrt{\frac{(h_0-h_2)/h_0}{\Psi+1-(h_2/h_0)^2}} \tag{7}$$

For free efflux $h_2 = C_c b$ and, if $H_L = 0$, Eq. (7) reduces to Eq. (2).

If Eqs. (3), (4), and (5) are solved simultaneously, an expression for the head-loss parameter can be obtained:

$$\Psi = \left(\frac{1}{\beta}\right)^2 + \frac{\gamma^2(\beta-1)}{\gamma\beta-2(\alpha-1)\pm\sqrt{[2(\alpha-1)-\gamma\beta]^2-\gamma^2(\beta^2-1)}} - 1 \tag{8}$$

In this equation the Greek-letter symbols are introduced for typographical reasons only and are defined as follows:

$$\alpha = \frac{h_2}{C_c b}, \quad \beta = \frac{h_0}{h_2}, \quad \gamma = \left(\frac{h_2}{C_c b}\right)^2 - \left(\frac{h_2}{h_0}\right)^2$$

A plot of Eq. (8) is shown on Fig. 6 as dashed curves. These curves, for different values of h_2/h_0 , can be seen to branch off from a common base line, the locus of the equation

$$\frac{H_L}{V_2^2/2g} = \left(\frac{y_2}{y_1}\right)^2 - 4\,\frac{(y_2/y_1)^2 - (y_2/y_1)}{(y_2/y_1)+1} - 1 \qquad (9)$$

which pertains to the hydraulic jump. Equation (9) is derived in a manner similar to Eq. (8). The depths y_1 and y_2 are equivalent to the depths $C_c b$ and h_2,

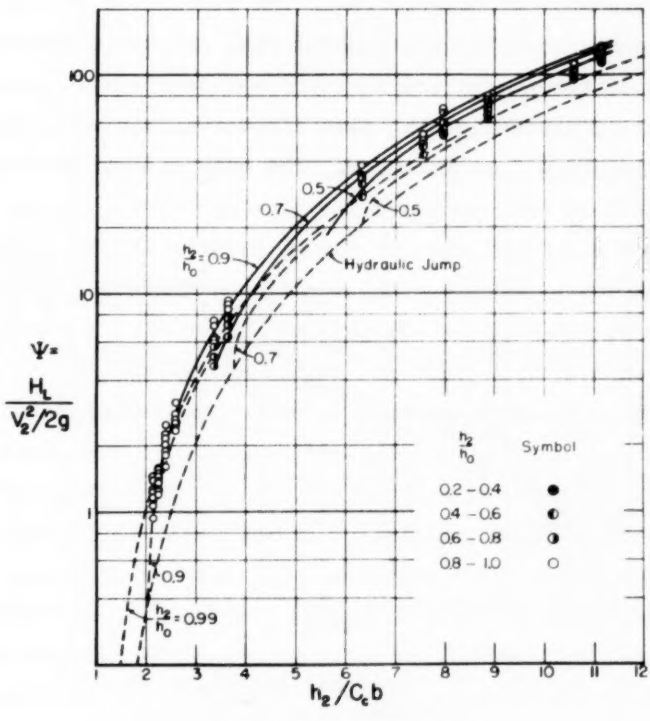

Fig. 6. Headloss parameter Ψ for submerged efflux

respectively, for the Tainter gate if the jump occurs at the vena contracta.

Data obtained in the second series of experiments were analyzed in the manner indicated by the theoretical

development and the results also plotted in Fig. 6. Because the geometry of the submerged jet could not be directly determined, the coefficient of contraction was assumed to be the same as for free efflux, although this is not strictly true. For a known lip angle, C_c was obtained from a mean curve through the plotted points in Fig. 5. All other values are based on directly measured quantities for submerged flow. The similarity in trend between the analytic and experimental curves is evident as is the difference in their respective locations. For a clearer comparison of the simplified analysis with experiment, Fig. 7 is included. The only difference between

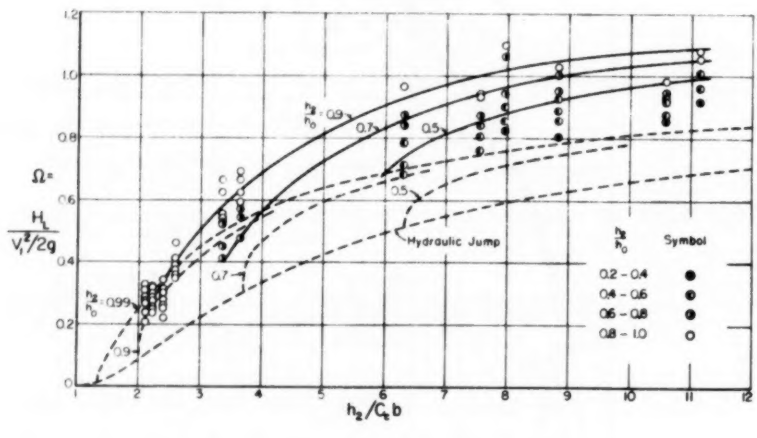

Fig. 7. Headloss parameter Ω
for submerged efflux

Figs. 6 and 7 is the use of the jet velocity head instead of the downstream velocity head in the head-loss parameter $\Omega = H_L/(V_1^2/2g)$. This change, easily accomplished numerically by multiplication of Ψ by $(C_c b/h_2)^2$, makes the plot more significant, mainly because the new head-loss parameter can be considered to show the energy loss

as a decimal fraction of the energy of the jet. The dashed lines of Fig. 7 which again are analytic results, show the necessary theoretical tendency for a limit of $\Omega = 1$ at large values of $h_2/C_c b$. For the experimental points, and the curves derived from them, the same general shape can again be noted, but the limit does not apply.

The lack of agreement in location between the analytically determined curves and those derived from experimental data stems from Eqs. (4) and (5) - some assumptions were made which are known to be at best merely approximations of the actual efflux from gates, but which were desirable because of the resultant mathematical simplicity. The square of the mean velocity was thus used as an indication of the kinetic energy in each section although the mean of the squares of the actual velocity distribution may well be considerably larger. The effect of boundary shear was completely ignored as were the complex flow conditions existing at section 1. Here the assumption of hydrostatic pressure distribution and the use of the mean velocity of the jet may be only very rough approximations of the mechanics of diffusion and of the roller above the jet. Equations (4) and (5) would have to be expressed as integrals [11], therefore, if a more accurate solution were required. For the purpose of this paper the simplified analysis is sufficient, however, because the important parameters were easily discernible, and the functional trend of the equations as shown in Figs. 6 and 7 would not be greatly changed by the refinements mentioned.

Closure

Discharge characteristics of Tainter gates, which heretofore were usually determined only for individual

gates, are shown to yield to functional representation for both free and submerged efflux. The systematic graphs, presented by Metzler, of the relationship between the discharge coefficient and the geometry of the gate and flow have been extended to include a variable trunnion height. Further extension of these graphs would permit representation of any desired geometry including piers and sills. Such extension would not affect the simplicity of calculating the discharge.

Although a complete theoretical analysis of the discharge characteristics does not seem possible at present, the existing theoretical tools can be applied to ascertain the important parameters and the functional trends among them. Under conditions of free efflux von Mises' contraction coefficient for slots with inclined walls is thus seen to approximate the variation with the lip angle of the coefficient for actual Tainter gates. The effect of the ratio of gate opening to headwater depth is indicated, moreover, by Pajer's analysis of the sluice gate.

A one-dimensional analysis of the flow similarly provides not only the parameters pertinent to the effect of submergence but also the nature of the functional relationships among them. In essence this analysis separates the effect of submergence from that of contraction. The latter must be assumed at present to be the same for submerged as for free efflux. For any jet geometry, however, a single family of curves should be sufficient to determine the head-loss parameters.

The results of the model studies presented in this paper are directly applicable to Tainter-gate installations on level floors. Their reliability, of course,

would be enhanced by comparison with prototype data because the effect of lip shape and of viscous forces were not determined in the models. The general methods of analysis applied to the data is applicable, however, to other types of gates.

Acknowledgments

The writer wishes to express his gratitude for their encouragement and advice to Dr. Hunter Rouse and Mr. Emmett M. Laursen, Director and Research Engineer, respectively, at the Iowa Institute of Hydraulic Research, and to Professor Donald E. Metzler, Research Engineer, for permission to use the data he had collected in his investigation.

References

1. Mises, R. von, "Berechnung von Ausfluss- und Ueberflusszahlen," Zeitschrift VDI, 1917, p. 47.

2. Pajer, G., "Ueber den Stroemungsvorgang an einer unterstroemten scharfkantigen Planschuetze," Zeit. Angew. Math. Mech., vol. 17, no. 5, 1937, p. 259.

3. Birkhoff, G., Hydrodynamics, Princeton University Press, 1950.

4. Jaeger, Ch., Technische Hydraulik, Basel, Birkhaeuser, 1949, p. 146.

5. Horton, R. E., "Discharge Coefficients for Tainter Gates," Engineering News-Record, Jan. 4, 1934, p. 10.

6. Koch-Carstanjen, Bewegung des Wassers und dabei auftretende Kraefte, Berlin, Julius Springer, 1926.

7. Metzler, D. E., "A Model Study of Tainter-Gate Operations," M. S. Thesis, State University of Iowa, 1948.

8. Laursen, E. M., "Model Studies Aid in Design of Brazilian Hydro Project," _Civil Engineering_, Jan. 1951.

9. Toch, A., "The Effect of Lip Angle upon Flow under a Tainter Gate," M. S. Thesis, State University of Iowa, 1952.

10. _Engineering Hydraulics_, edited by Hunter Rouse, New York, John Wiley and Sons, Inc., 1950, p. 540.

11. Rouse, H., _Elementary Mechanics of Fluids_, New York, John Wiley and Sons, Inc., 1946, pp. 111-113.